## THE CONSTRUCTION OF HANDICAP TABLES FOR ARCHERS

## 1 Introduction.

Before setting about the construction of handicap tables for archers, it is worth considering why they should be constructed at all. The usual answer - that they allow archers of different standards to compete against each other - is one that I personally find inadequate. In the end, the top competitions are all F.I.T.A., rounds and the winner is judged on an absolute basis - the archer with the highest score wins. The aim of handicap tables therefore should be to teach the archer something about his own skill and to provide an absolute measure of skill. Practice by shooting complete F.I.T.A. rounds is not always possible and in the U.K. we also have the complication of the traditional English rounds. We also need to consider the junior archer, shooting shorter range rounds. Obviously we need a way which will allow them to judge whether their basic skill is improving as they move to heavier bows and longer ranges.

Thus, in my view, we need a set of tables based on a fundamental measure of skill which can be applied to all the different ranges, target sizes and scoring methods and allow the archer to know whether he is shooting better or worse and exactly how much better or worse.

Having taken this approach, it seems to me that the angular- dispersion of shots provides the fundamental measure of skill which is required and it is this basis that has been used in constructing the handicap tables.

## 2 General Theory.

### 2.1 The Distribution of Shots.

The archer aims to hit the pin-hole on the target, but due to the cumulative effects of many small variations and errors, the shots are distributed round the pinhole. The distribution of shots conforms to the general pattern of a circular binormal distribution. Thus the distribution of miss distances measured from the pinhole will conform to the Rayleigh distribution and the probability density function is:-

$$
p(r)=\frac{2 r}{\sigma_{r}^{2}} \exp \left(\frac{-r^{2}}{\sigma_{r}^{2}}\right)
$$

where r is the miss distance from the pinhole and $\sigma_{r}$ is the RMS miss distance.
Thus the average score of each arrow is:-

$$
\begin{equation*}
S=\int_{0}^{\infty} S(r) \cdot \frac{2 r}{\sigma_{r}^{2}} \cdot \exp \left(\frac{-r^{2}}{\sigma_{r}^{2}}\right) \cdot d r \tag{1}
\end{equation*}
$$

Where $S(r)$ is the staircase function representing the particular scoring system on the particular target size in use.

Thus if D is the diameter of the target, then for metric rounds :-

$$
S(r)=10 \text { for } 0<r<\frac{D}{20}
$$

$$
\begin{aligned}
& =9 \text { for } \frac{D}{20}<r<\frac{2 D}{20} \\
& =8 \text { for } \frac{2 D}{20}<r<\frac{3 D}{20} \quad \text { etc. }
\end{aligned}
$$

While for Imperial rounds:-

$$
\begin{aligned}
S(r) & =9 \text { for } 0<r<\frac{D}{10} \\
& =7 \text { for } \frac{D}{10}<r<\frac{2 D}{10} \quad \text { etc. }
\end{aligned}
$$

Thus S may be evaluated as a function of $\sigma_{r}$ for all values of D in normal use i.e. $122 \mathrm{cms}, 80 \mathrm{cms}, 60 \mathrm{cms} 40 \mathrm{cms}$ and for Metric and Imperial methods of scoring.

The arithmetic of the integration is tedious since it has to be carried out over 10 or 5 intervals, but fortunately with the aid of modern computers we do not have to worry about this aspect.

### 2.2 The Line Cutter Rule

Before proceeding any further it is worth noting that the effect of the thickness of arrow which is used and the present line cutter rule can be taken into account simply by increasing the size of each target zone by the radius of the arrow.

### 2.3 Angular Accuracy

Equation (1) overleaf would be sufficient for our purpose if targets were always at the same range. In this case the RMS miss distance $\sigma_{r}$ would be the fundamental measure of skill which we require. However, it is obviously necessary to extend the analysis to incorporate the effect of targets at different ranges.

In order to do so we write:- $\sigma_{r}=R \sigma_{\theta}$ where $R$ is the range to the target and $\sigma_{\theta}$ is the RMS angular error achieved by the archer.

Thus the average score of each arrow is given by:

$$
\begin{equation*}
S=\int_{0}^{\infty} S(r) \cdot \frac{2 r}{R^{2} \sigma_{\theta}^{2}} \cdot \exp \left(\frac{-r^{2}}{R^{2} \sigma_{\theta}^{2}}\right) \cdot d r \tag{2}
\end{equation*}
$$

and $S$ may be computed as a function of $\sigma_{\theta}$ for whatever ranges, target sizes, scoring methods we wish.

Thus $\sigma_{\theta}$ - the RMS angular dispersion achieved by the archer - appears to offer the fundamental measure of skill -for which we are looking. As long as an archer is reducing the RMS angular dispersion, skill is improving and equation (2) above offers us the way of measuring skill that is independent of range, target size or scoring method.

Unfortunately, it is not quite as easy as that. Because of finite arrow velocity and air retardation $\sigma_{\theta}$ is itself dependent on range i.e. an archer shooting with exactly the same skill will not achieve the same angular dispersion at 90 m as at 30 m or 10 m and we need to extend the analysis one stage further.

### 2.4 The Effect of Range on Angular Dispersion.

As noted above, because the arrow is shot with a constant finite velocity, elevation has to be increased as the range to the target increases and the .arrow spends more time in the air than would be expected simply by scaling proportional to target range. The effect of air retardation increases this effect and the total result is that because of the curvature of the trajectories, the apparent angular dispersion increases as target range increases.

Two alternatives offer themselves. This effect could be estimated theoretically by considering the detailed ballistics of arrow trajectories, taking into account air resistance and typical velocities. Alternatively, the average degradation of angular accuracy as a function of range may be estimated from tournament results.

Both methods are beset with difficulties. The first alternative involves calculation of trajectories subject to air retardation which is inherently difficult and also subject to error because of lack of adequate data on the drag coefficients of arrows. The second is subject to all the statistical problems of different tournament conditions, archers having a good 90 m followed by a bad 70m, large amounts of tedious analysis, etc., etc.

Nevertheless, in spite of the statistical difficulties, the second alternative was chosen since the result, if it could be achieved, would reflect actual performance in tournament conditions whereas the former might be criticised on the basis of being a paper study remote from reality.

## 3 Analysis of Tournament Results

Preliminary calculations were carried out using Equation (2) to tabulate average score as a function of angular accuracy for all normal target sizes, ranges and scoring methods.

Tournament results which listed scores at each separate range were then analysed to convert each archer's score at each range into the angular accuracy which had been achieved.

The tournaments analysed in this way included the Olympics, the 1977 World Championships, the 1977 Junior F.I.T.A. Mail Match, and the 1976 Junior National Championships and Junior FITA Mail Match.

A selection of the individual archer's results is shown in Fig. 1.
Examination of these results suggested that the way in which angular dispersion varies with range could be represented by:-

$$
\sigma_{\theta}(R)=\sigma_{\theta}(30)[k(R-30)+1]
$$

where $\sigma_{\theta}(R)$ is the angular dispersion at range $R$
$\sigma_{\theta}(30)$ is the angular dispersion at 30 m
and $\quad k$ is a constant.
Lines of this form are also shown in Fig. 1 to indicate how they fit the tournament results.

Thus $\quad F(R)=k(R-30)+1$
where $F(R)=\frac{\sigma_{\theta}(R)}{\sigma_{\theta}(30)}$ and may be determined from each archer's tournament result.

The best value of ' $k$ ' was determined for each tournament by the method of least squares. As might be expected, there is a variation of the value of the constant ' $k$ ' from one tournament to another. But apart from this variation it is important to see whether there are differences for ladies, gentlemen, girls, boys. If such differences exist and are significant then it would be virtually impossible to achieve a comprehensive set of handicap tables. Separate tables would have to be computed using the appropriate value of ' $k$ ' in each case.

However, inspection of Fig. 1 shows that using a single value of ' $k$ ' gives a set of straight lines which is a reasonable fit to all of the results, including such widely varying standards from the Olympic and World Championship archers down to the Under 12's shooting in a National Championship.

The use of a single value of ' $k$ ' is discussed further in a later section of this note.

## 4 Handicap Tables

Having determined the value of the constant ' $k$ ' i.e. the way in which angular accuracy varies with range and on the assumption that this single value can be used for all classes of archers, we can now compute the scores to be expected for any round from the expressions:-

$$
T=\sum_{n} N_{n} \int_{0}^{\infty} S_{n(r)} \cdot \frac{2 r}{R_{n}^{2} \sigma_{\theta}^{2}\left(R_{n}\right)} \cdot \exp \left(\frac{-r^{2}}{R_{n}^{2} \sigma_{\theta}^{2}\left(R_{n}\right)}\right) \cdot d r
$$

And $\quad \sigma_{\theta}\left(R_{n}\right)=\sigma_{\theta}(30)\left[k\left(R_{n}-30\right)+1\right]$
where $T$ is the total score for the round,
$N_{n}$ is the number of arrows shot at range $R_{n}$,
$S_{n}(r)$ is the staircase function for the size of target used at range $R_{n}$ and the scoring method, and the summation is carried out over the four or three ranges shot in each round.

While these expressions are cumbersome and their evaluation involves an enormous amount of arithmetic, this represents no real problem since it has all been programmed and carried out on a computer.

Thus the total for each round is a function of $\sigma_{\theta}(30)$ and can be tabulated in terms of $\sigma_{\theta}(30)$. Scores for 2 or 3 dozen arrows at single ranges can obviously be included in the tabulation.

The important features of this analysis are fairly simple and it is worth stating them clearly:-
(a) The angular dispersion (RMS) at a single standard range (30m) is a fundamental measure of the archer's skill.
(b) On average, the angular dispersion at any other range follows automatically from the dispersion achieved at 30 m . The same law appears to apply to both sexes and all age groups. The variations are more dependent on tournament and weather conditions than they are on age or sex.
(c) The scores for all rounds which may be shot can therefore be tabulated in terms of the angular dispersion achieved at a standard range ( 30 m ) and such tabulations obviously provide handicap tables.
(d) The handicap tables computed on this basis satisfy the requirements defined in the introduction to this note. Most importantly, they allow the archer to measure the performance achieved in order to see whether any special range or condition is particularly weak or strong.

## 5 Detailed Discussion of Results.

### 5.1 General Tabulations

Figure 2 shows results calculated from Equation 2. The column headed S gives the value of the RMS angular dispersion $\sigma_{\theta}$ in milliradians which has been used for each line of results. It should be noted that in these preliminary results that $\sigma_{\theta}$ is kept constant for all ranges in each line of results.

The tabulation was started at a value of $\sigma_{\theta}$ of 0.5 mrad . This value was chosen since it represents the average limiting value of the resolution of the human eye. It is interesting to note that if this value of dispersion could be achieved in practice, near maximum scores would be shot with the current sizes of targets at all ranges out to 90 m and 100 yds . It is obviously therefore, an adequate starting point.

Preliminary calculations also showed that the range of angular dispersions which needs to be covered is from 0.5 mrad . to about 25 mrad at which value a score of only 36 is being achieved for 3 dozen arrows at 30 m on an 80 cm metric face.

The range from 0.5 mrad to 25 mrad could have been divided into say 100 equal steps of 0.25 mrad. However it was felt intuitively that such a scheme does not correspond to equally achievable increments of performance. Thus it is comparatively easy for a beginner to reduce angular dispersion from say 10 mrad to 9.75 mrad , but it is extremely difficult to reduce dispersion from 1 mrad to 0.75 mrad . An. alternative scheme was therefore chosen based on a constant percentage increment.

The same range can be covered with 100 steps each of $4 \%$. The choice of $4 \%$ also commends itself because it equates roughly with the standard deviation to be expected due to the statistics of sampling associated with a single tournament result as opposed to the average of many tournament results. Thus a $4 \%$ increase in performance (i.e. reduction of angular dispersion) may be regarded as the minimum significant step which should be recognised in terms of handicap improvement. Smaller improvements may well be due to random sampling effects rather than any increase in skill.

Thus in the computed tables the angular dispersion for each line has been taken to be:-

$$
\begin{equation*}
\sigma_{\theta}(N)=0.5 \cdot(1.04)^{N} \quad \mathrm{mrads} \tag{4}
\end{equation*}
$$

where N may be taken as a handicap rating number and the range of $\sigma_{\theta}$ covered is given by $\mathrm{N}=0$ (1) 100 .

### 5.2 Effect of Arrow Size

The tables illustrated in Figure 2 were calculated using 16/64, 18/64 and 20/64 arrow sizes to see whether the use of larger arrows produces significantly higher scores. Figure 2 includes examples of the differences at different values of angular dispersion $\sigma_{\theta}$. It will be seen that moving from 16/64 arrows to 20/64 arrows increases the score by a maximum of about 3 to 4 points in a full round. As a result, all subsequent calculations have used 18/64 arrows and separate results for other arrow sizes have been dropped.

### 5.3 Analysis of Tournament Results

The analysis of tournament results is illustrated in greater detail in Fig 3. Using the basic tables referred to in the previous section, the scores of each archer at each separate range are translated into angular dispersions. Using the shortest range as the standard ( 30 m generally), the ratios $F(R)=\frac{\sigma_{\theta}(R)}{\sigma_{\theta}(30)}$ are calculated for each archer.

Using the method of least squares, a straight line of the form
$F(R)=k(R-30)+1$ was fitted to each tournament result and the best value of the constant ' $k$ ' determined.

The values of ' $k$ ' so determined are listed in the following table:-

| Tournament | Class | No. Competitors | ' k ' per metre |
| :---: | :---: | :---: | :---: |
| 1976 Olympics | Ladies | 2x27 | 6.807(-3) |
|  | Men | 2x37 | 7.154(-3) |
| 1976 JFMM | Girls | 10 | 5.976(-3) |
|  | Boys | 53 | 4.521(-3) |
| 1976 Junior Nat. Champ. | Girls<18 | 6 | -4.602(-3) |
|  | Girls<16 | 10 | 0.393(-3) |
|  | Girls<13 | 8 | 8.772(-3) |
|  | Boys<18 | 13 | 7.410(-3) |
|  | Boys<16 | 25 | -0.870(-3) |
|  | Boys<14 | 20 | 3.876(-3) |
|  | Boys<12 | 7 | 1.057(-3) |
| 1977 World Champs. | Ladies | 2x43 | 2.759(-3) |
|  | Men | 2x66 | 3.411(-3) |
| 1977 JFMM | Girls | 17 | 5.586(-3) |
|  | Boys | 40 | 5.179(-3) |
|  |  | Weighted Average | 4.310(-3) |

It should be noted that the value of ' $k$ ' for an imperial tournament is basically calculated as the slope of the line per yard of range to the target. In order to avoid confusion of units, the values in the table above have been converted to the slope per metre of range to the target in order to line up with the metric tournaments.

Examination of these results provides many areas for discussion and future analysis. The conclusions are necessarily limited by the small number of tournaments which have been analysed. This analysis is undoubtedly tedious and time consuming since it has not yet been possible to programme it for a computer and even if it had been, it would require all of the individual score data for each archer to be punched up as input data for the computer. Therefore, all of this analysis has been carried out by hand.

It may also be noted that this type of analysis requires a tournament results sheet which lists the scores for each range separately and the number of such results sheets available to the author is limited. There is also a regrettable tendency for tournament organisers to publish only total scores, which precludes this type of analysis.

Nevertheless, in spite of the limitations, it is felt that the results are worthy of examination.

It must be remembered that the 1976 Olympics and the 1977 World Championship include a large number of archers shooting in both tournaments and therefore, we have the chance of comparing the same archers shooting in different tournament conditions.

It will be seen that for the men the values of ' $k$ ' are $7.154(-3)$ and $3.411(-3)$ while for the ladies the values are $6.807(-3)$ and $2.759(-3)$. Thus there is considerable variation between one tournament and another, but the values for the men and ladies overlap.

The results for the JFMM's are $5.976(-3), 5.586(-3)$ for girls and $4.521(-3), 5.179(-$ 3 ) for boys. All values are within the total range of results from the Olympics and World Championships.

The values for Juniors show considerable variation and even, in two cases, negative values - indicating that accuracy gets worse as the target gets nearer. This might indicate that tiredness is adversely affecting their accuracy as the tournament proceeds. However, the numbers of archers in the various junior classes is small and obviously limit the accuracy of the results. Overall, it is felt that there is too little evidence of correlation between values of ' $k$ ' and different classes of archers to justify the use of more than one value in a set of handicap tables, particularly in the light of the obvious variation between one tournament and another, even with largely the same archers competing.

To improve this situation, refine the value of ' $k$ ' , and to examine in more detail whether there is correlation with age or sex, will require the analysis of many more tournament results and it is strongly recommended that this analysis should be carried out. But in spite of this, it is concluded that the incorporation of the average value of ' $k$ ' so far determined, into the computation of handicap tables would provide a set of tables applicable to all classes of archers which would be significantly more realistic, consistent and useful than the present sets of tables.

Thus in the set of Comprehensive Handicap Tables which have been produced and are attached as an Annex to this note, a value of $\mathrm{k}=4.3(-3)$ per metre has been used in Equation 3 and all subsequent calculations.

## 6 The Handicap Rating Number.

In paragraph 5.1 of this note, the number N of $4 \%$ steps required to increase the angular dispersion from 0.5 m rad to any other value was referred to as a Handicap Rating Number and all of the calculations were carried out using values of N from 0 to 100 .

Traditionally, Handicap Ratings are both positive and negative with the zero rating referred to as "scratch", and there appears to be a great deal of emotional attachment to the idea of a scratch rating attached to well-known score levels i.e. 800 for a York for men and for a Hereford for ladies.

Basically it appears to have escaped notice that these scores do not represent the same level of skill. 800 for the Hereford implies an angular dispersion which is $35 \%$ worse than 800 for the York.

This represents a problem for a set of comprehensive tables applicable to ladies, men, girls and boys since obviously the scratch rating can only be set at one level in the tables. The complication of providing two or more handicap rating numbers (one for men, one for ladies, etc.) in the same set of tables tends to defeat the whole object of making the tables a basic measure of skill. In the present tables, therefore, the handicap rating number has been arbitrarily set so that the scratch rating equates to a score of 800 on the York round.

In practice this meant putting $\mathrm{H}=\mathrm{N}-47.5$ where N is the index used in equation (4) and H is the handicap rating number.

## 7 Comprehensive Handicap Tables.

Using the principles and the tournament analysis results which have been described in this note, a set of handicap tables were computed and are attached as Annex 1.

Examination, of these tables will show that they are totally consistent and free from all of the limitations arid inconsistencies present in the current sets of tables

Looking to the future, it is worth considering what circumstances would require revision of the tables. Two such eventualities can be foreseen. Firstly, it is clear that because near maximum scores are already being shot at 30 m it may be necessary in the fairly near future to reduce the size of target used at 30 m to say a 60 cm target rather than an 80 cm target. Such a change would involve a minor data change in the computer programme and a re-run. The problems therefore would be associated entirely with printing and distribution and would not involve any reconsideration of the principles on which the tables are based

The second eventuality which may occur is that further analysis of tournament results will show that the degradation of accuracy with range (represented by the constant ' $k$ ' ) requires modification. Equipment development over the next few years may also modify this law. This is a situation which should be kept under constant review. However, it can be noted that the absolute value of ' $k$ ' which is used in the computation has only a small effect on the total scores. Thus even if changes are necessary, they are unlikely to occur very frequently.

## 8 Conclusions.

The basic theory set out simply reflects standard statistical theory. The incorporation of the results of tournament analysis ensures that the theory is linked positively and realistically to the current performance of archers and equipment.

The Handicap Tables produced on the basis of this analysis are consistent and provide the archer with an accurate, absolute measure of skill.

This absolute measure of skill applies to both sexes and all age groups and the use of the Tables will obviate all of the inconsistencies and confusion associated with the present mixture of Junior and Senior Tables.

The need for frequent revision, which with the present tables occurs mainly because of the erroneous basis on which the Junior Tables have been constructed, will be avoided.

## 9 Recommendations.

Apart from the obvious need to get the proposed tables ratified and into common use as quickly as possible, there are further recommendations which need to be emphasized. The first is for tournament organisers to show the scores for each range separately in the Results. Without this, any sensible analysis of tournament results is impossible.

The second is that analysis of the results along the lines suggested in Section 3 of this note should be organized on a routine basis.

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15.3.78




Seos $185 \quad 241 \quad 254 \quad 303$
0ө $4.204 \quad 3.638 \quad 3.184 \quad 3.254 \quad 1.292 \quad 1.118 \quad 0.978$
इeote $233 \quad 259 \quad 244 \quad 303$
$\begin{array}{lllllllllllllllllll}50 & 3.135 & 3.311 & 3.444 & 3.254 & 0.963 & 1.018 & 1.058\end{array}$
Scote $195 \quad 206 \quad 233 \quad 292$
$\begin{array}{lllllllllllllllll}\sigma 5 & 3.975 & 4.783 & 3.720 & 3.732 & 1.065 & 1.282 & 97\end{array}$
score $\begin{array}{llll}182 & 265 & 243 & 306\end{array}$
69 $4.276 \quad 3.135 \quad 3.471 \quad 3.159 \quad 1.354 \quad 0.992 \quad 1.099$

Reen thêe $1.2041 .197 \quad 0.949$

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